THE DURAND COMPUTUS
IN BOOK VIII RATIONALE DIVINORUM OFFICIORUM

Abstract
The main task of the study is to present the computational tools used by Durandus. The time of his scientific activity falls on the early period of computation development, and individual computists based their work on various elements of the computus, referring to Dionysius, Isidore of Seville, Bede the Venerable or Alexander of Villedieu (d. 1240). The aim of this study is also to analyse and diagnose the Durandus calculation algorithm and its originality in the computistic matter. Durandus’ study refers to the works of earlier computists, and its originality lies in the selection of content. Some chronological issues are presented extensively (such as a 19-year cycle or months). Others, however, are only signalled in his work (such as the issue of concurrents or claves terminorum, which needed to be developed in the content of the article). The concise concept of the ecclesiastical computus contained in the Rationale divinorum officiorum refers in its content to the findings of the computists of the Roman calculation of Easter. His treatise, although setting out the rules of conduct in liturgical matters, strengthens and popularizes the Roman system of Paschal calculus adopted from its predecessors, and also defines the Church computus as an important element of the functioning of canon law.

Keywords: Guillaume Durand; medieval chronology; computus; Paschal reckoning; liturgical time

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Computus as the basis of ecclesiastical chronology

Measuring and marking time using observations of astronomical phenomena began as early as ancient times, when the relationship between the periodic character of celestial and terrestrial phenomena was discovered. The consequence of these discoveries was the creation of the calendar, which is an instrument that organizes celestial phenomena in relation to the periodicity of human life. Since Christian antiquity, the measurement of time and its determination was based on observations of the rhythm of astronomical phenomena and their relation to the periodicity of human life. The measurement of time has also been a key element for the organization of the Church’s liturgy system from the beginning.

In the early Middle Ages, computistics, a field of chronological science, began to take shape on the ruins of the Western Roman empire, with the computus as its peak achievement. Computistics has been based on the lunisolar calendar since its inception, and the Durand computus, the subject of this study, is based on the same foundation. Computists throughout history have used various tools and techniques in marking the dates of the lunisolar calendar and the cycles of the lunisolar reckoning. The essential task of the study, therefore, is to present the computational tools used by Durandus. The time of his scholarly activity is in the early period of the development of computistics, and individual computists based their work on various elements (components) of the computus, referring to the pillars of computational calculus – Dionysius Exiguus (d. 544), Isidore of Seville (d. 636), Bede the Venerable (d. 735) or Alexander of Villedieu (d. 1240). Outside the canon – i.e., the 19-year and 28-year cycles – calculating its elements was done in a variety of ways, as each creator constructed his individual computational algorithm. The purpose of this paper is to analyse and diagnose the Durandus’ calculus algorithm and its originality in computistic matter.

The contents of the computus consisted of elements of theoretical and technical chronology that make up the connective tissue of the lunisolar calendar. Its chief task was to correlate the elements of the Julian solar calendar with the lunar calendar in order to mark for each year the calendar days of the week and month and the days of the lunar month that fall on them, as the modern poem puts it: ‘quota sit luna in kalendas quasque.’ The time reckoning cultivated at the time was based on the Alexandrian tables of the Lunisolar Paschal reckoning and the components of the solar Julian calendar.\footnote{The name ‘computistics’ originates from the Latin verb \textit{computare}, meaning to count, reckon, calculate. Computistics deals with chronology in technical or mathematical terms. See H. Gro\-tefend, \textit{Taschenbuch der Zeitrechnung des deutschen Mittelalters und der Neuzeit}, Hannover 1960; A. Borst, \textit{Computus – Zeit und Zahl in der Geschichte Europas}, Berlin 2004.}

\footnote{The Julian year was 11 days longer than the lunar year and hence the age of the moon on 1 January each year is 11 days greater. When it reaches the number of 30 days, a leap month is inserted and the year then has 13 months. The difference in days between the calendar year and the lunar year was called epacts. By epacts, Augustalis meant the age of the moon on 1 January. They were 1 in the first year of the cycle, 12 in the second, 23 in the third, and so on.}
Alexandrian counting rules were translated into the months of the Julian calendar and the years of the lunar cycle in the first half of the 6th century, by Dionysius Exiguus (d. 523), who created a Latin version of the Alexandrian Computus and presented it in the form of Paschal tables for 95 years. In his commentary on these tables, titled Argumenta paschalia, Dionysius included the first detailed description of the algorithm of operations performed in computational calculations. The author explained the rules of construction of the computus and the procedure for determining the various fixed components of the lunisolar calendar, used in the Paschal tables to calculate the dates of Easter.

Simple and abbreviated descriptions of the chronological algorithm, gradually enriched with new and more detailed content, began to take on more extensive forms in the early middle ages. In addition to the description of detailed calculations, they were accompanied by numerous mnemonic lines and, over time, acquired the characteristics of a remarkably specialized and intricate calculation apparatus. In computistics textbooks, authors gave a display of their chronological erudition, sometimes going beyond the role of mentor and establishing a bond with the reader in the form of an intellectual game with them. Computus, initially present in the basic Dionysian form and implemented on the few components of the lunisolar calendar, in later textbooks takes the form of a sophisticated chronological apparatus, filled with explanations of the increasingly numerous elements of time reckoning. As the volume of the content of the treatises increased, the matter pertaining to chronological calculations began to be referred to by its creators as massa compoti.

The continuator and promoter of the Dionysian reckoning of time was Bede the Venerable, whose computus, known throughout the medieval world and his primary research problem: ‘Quota sit luna in Kalendas quasque; quae sit feria in Kalendas,’ acquired the characteristics of a chronological chronogram, and in this way Western computistics turned into mathematical and astronomical knowledge, based on the use of the periodicity of celestial phenomena. It is particularly specialized knowledge, intended for the use of a few, for those who, using only the constants of the lunisolar calendar, are themselves, without external assistance, able to calculate, for any year, the relationship of the days of the week to the days of the month of the civil calendar and the corresponding days of the lunar year, and consequently the dates of the movable holidays headed by Easter. In order


4 The first chapter of Dionysius Exiguus’ commentary (translated by Naumowicz, Geneza, p. 247): ‘If you want to know what year passes since the incarnation of our Lord Jesus Christ, multiply 15 by 34, which gives 510. To this add always 12 as a fixed number, you get 522. You also have to add the indiction of a given year, for example, the third one that falls during the consulship of Probus the Younger, making a total of 525. Such is the year since the Lord’s incarnation.’

5 Bede the Venerable, De temporum ratione, cap. XX, XXI.
to transmit this knowledge to a wider range of users, textbooks on the art of time reckoning were created, where the dates of the components of the lunisolar cycle were given in the form of chronograms and mnemonic notations.

The Julian calendar and the 19-year lunar cycle were enshrined in Church doctrine and therefore became part of canon law. As such, they are an integral part of the liturgical books, and the key building blocks of the lunisolar cycle.\(^6\)

The basic tool for determining the chronological reckoning component is mainly based on the observation of the cyclicity of the course of the Sun and Moon, using a 28-year and 19-year cycle, respectively. Based on this data, it is possible to unerringly determine the date of Easter, and thus predict the time frame of the entire liturgical year. Thus, a system of measuring time derived from the rhythm of the liturgical year was developed, and it can even be assumed that this rhythm accurately regulated the entirety of social life. In the matter of medieval time reckoning, at least two layers can be distinguished – ecclesiastical and scholarly, which perfectly blended with each other, assimilating the sacrum and profanum, and thus binding the elements of the physical and ecclesiastical calendar into an indivisible whole.\(^7\) The computus, therefore, as a method of reckoning time centred on calculating the date of the most important feast of the Roman Catholic Church, is the canon of the liturgy of the Church and the foundation for its functioning.

**The author and his work**

The Middle Ages did not develop a universal reckoning of time. In addition to the Roman calendar and the solar cycle, the lunar calendar and the lunar cycle were used. Both of these formed the structural basis of the lunisolar calendar and the paschal computus and chronological art (\textit{ars computandi}) based on it, used to determine the dates of the movable feasts with Easter at the forefront.\(^8\) The emerging numerous computistic works framed the ancient \textit{ars computandi} with additional content aimed at improving the lunisolar calculus. Furthermore, in order to raise awareness of the usefulness of the knowledge they conveyed, they began to be placed within the structure of more extensive treatises on the subject of the Church liturgy. One of the scholars who chose this particular method of compilation and presentation of computational knowledge was Guillaume Durand, also known as Durandus (1230–1296), a French scholar, bishop of Mende, canonist, liturgist and, apparently, a computist, although the latter profession stems, as it were, from his interest in liturgy. The scholar is at the centre of this science, among other reasons, because he was the first to include computi in church legislation, making it one of the elements of the \textit{divinum officium}. He included a concise version of

\(^6\) Regulations on time reckoning are also present in the contemporary Code of Canon Law, see \textit{Codex Iuris Canonici auctoritate Ioannis Pauli PP. II promulgatus} (25.01.1983), Acta Apostolicae Sedis, 75 (1983), pars I, t. XI, Cann. 200-203, p. 32; Polish text in: \textit{Kodeks prawa kanonicznego}, Poznań 1984.


\(^8\) For more on time reckoning in the Middle Ages, see J. Syty, \textit{Oznaczanie czasu w średniowiecznych źródłach narracyjnych}, Roczniki Humanistyczne, 33 (1985) issue 2, pp. 5–47.
the textbook of the ecclesiastical computus in book VIII of the treatise *Rationale divinorum officiorum*. As agreed by medieval scholars, the creator of the *Rationale* is unquestionably one of the most famous experts of late medieval liturgy. Despite his comprehensive research interests and extensive writing, he remains little known in the Polish scientific community. His name is mainly associated with liturgical writings and reflections on a universalist vision of culture and the world.

G. Durand was born around 1230 in Puimisson, but there is no information about his family or upbringing period. It is known that he was ordained a priest in 1254, which formally launched his scientific career. He began his education at a cathedral school in Provence and graduated it with a doctorate in canon law at the University of Bologna around 1263. He was a lecturer in canon law for several years and then, after being appointed by Pope Gregory X, moved to Rome and began working in the papal administration. This career was interrupted by his election as bishop in Mende, France (1285). He devoted much of his time as bishop to work on liturgical treatises that established his reputation as the most outstanding liturgist of the time. Among other works, *Liber Ordinarium*, the liturgical regulations for worship at Mende Cathedral, was written at the same time. In addition, Durand published *Constitutiones synodales* and *Instructiones*, the first synodal statutes in his diocese. He also completed the treatise *Pontificale*, which was then de facto taken over by the Vatican as the papal pontifical.\(^9\) Also during his episcopate in Mende, the liturgist completed his most elaborated and at the same time most famous work in the field – *Rationale divinorum officiorum*, the text of which did not take its final form until its second redaction (1294–1296).\(^10\)

The comprehensive treatise, which combines encyclopaedic features with allegory, is the best known work on liturgy of the time, described by specialists in liturgy and worship as late as the 19th century as a complete work, expressing the entire theory and reflection on the liturgy of the Church of the late Middle Ages.\(^11\) The treatise provides an extensive and exhaustive overview of the issues and areas of functioning of the Roman Church and its liturgy, ranging from architecture and sacred art to instructions for the liturgical service to the ecclesiastical computus and liturgical calendar. The purpose of the treatise is illustrated by one of the elements of its title: *rationale* means that it is a set of laws, a kind of model for liturgical activities within the Mende diocese administered by the author.

Referring to the richness of the content of the *Rationale divinorum officiorum*, it should also be noted that the work cannot be regarded as a new and revealing interpretation of the liturgy and ecclesiastical chronology of the 13th century. Rather, it should be said that the author has compiled and summarized the work of earlier exeges of the liturgy: Amalarius of Metz (d. 850), Honorius of Autun (d. 1140) or William of Auxerre (d. 1231).\(^12\)

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9 Durand’s pontifical was used in the Vatican until the reforms of the Second Vatican Council.
The treatise is also a sort of voice in the discussion on the timing of Easter celebrations. Although the clashes between Rome and Alexandria over the date of this feast were a thing of the past in Durand’s time, one cannot help but agree that the affirmation of specific modes of Easter reckoning in a work that deals with issues of the liturgy in the broadest sense of the term, solidified and popularized them on European scale. Previously popular in Gaul from the mid-fourth century were the Paschal tables of Victorius of Aquitaine,\(^\text{13}\) which reckoned the dates of Easter differently from the Alexandrians, but these tablets did not gain official approval from Rome precisely because of this incompatibility of reckoning.\(^\text{14}\) The dispute finally came to an end in the 6th century, when the Paschal tables of Dionysius Exiguus were adopted as the official system of reckoning and made those of Victorius obsolete.\(^\text{15}\) Durand’s treatise is thus part of the history of the development of French liturgical and chronological thought, and at the same time confirms the reception of the Dionysian reckoning system north of the Alps.

**Time and computus for Durandus**

The basis for the treatise’s indications of liturgical practice and worship are the computational calculations and regulations, which the author particularly highlighted in the work’s Book VIII, entitled *De compoto et calendario et de pertinentibus ad illa*. This part of the treatise includes not only the definition of the computus and the calendar, but also presents considerations of time, methods of determining it and algorithms for calculating its various components. Situating the issue of the ecclesiastical calendar in the last, eighth book of the treatise seems to be a deliberate move by the author, who indicates that all areas of Church activity described in the earlier books intertwine on the plane of ecclesiastical chronology. The rhythm of the Church life is regulated by the guidelines of the liturgical calendar and vice versa – the Church’s various activities implement the indications of the liturgical calendar. Thus, the coexistence, as well as the interdependence of spatial and temporal realities is emphasized. By organizing the contents of Book VIII of the treatise into purely astronomical and computational ones, the author automatically divides his textbook into theoretical and practical parts. The theoretical part includes information on astronomical phenomena and their translation into the time reckoning system, and the exercise part includes a description of the components and method of their calculation.

This reflection does not introduce a breakthrough into medievalist computistics, as it continues the thought of great predecessors – Bede the Venerable or Alexander of Villedieu. Nevertheless, the author’s work, as the most momentous of the liturgical writings of the period, unifies, consolidates and disseminates the basic knowledge of computistics. He also points out the versatility and functionality of

\(^\text{13}\) For the table of Victorius (*Cursus paschalis*), see Krusch, *Studien*, pp. 17–52.

\(^\text{14}\) For the methods of the Paschal reckoning of Victorius of Aquitaine and their reception, see Ch.W. Jones, *The Victorian and Dionysiac Paschal Tables in the West*, Speculum, 9 (1934) pp. 408–421.

\(^\text{15}\) For the dispute over the date on which Easter is celebrated and the related controversy, see J. Naumowicz, *Trzy spory o datę Wielkanocy*, Vox Patrum, 26 (2006) vol. 49, pp. 453–469.
chronometric components, especially the golden number and ferial letter, which occur in the liturgical books of the Roman Church.

‘Compotus est scientia certificandi tempus secundum solis et lune progressum (...) Dicitur autem compotus a computo-tas quia in ipso computando procedimus non quia in ipso computare doceamur.’16 The definition of the computus in Rationale is not fundamentally different from the one composed by Bede the Venerable.17 It is also worth juxtaposing its definition with the definition of the object and scope of this field of knowledge contained in the thought of the Bishop of Lincoln, and Oxford professor and distinguished computist, Robert Grosseteste (d. 1253). The scientist included a definition of this field at the beginning of his best-known work on the art of time reckoning: ‘Compotus est scientia numeracionis et divisionis temporum... per signationes quas dant eis motus celestium corporum (...) et (...) modus numerationis et divisionis temporum completa est scientia que Comptotus nominantur.’18 Both definitions indicate the close interdependence of knowledge of the computus with the observation of astronomical phenomena and the determination of time based on the structural units assigned to it. They are dichotomous in nature, and include linear and cyclical time. The idea is that they form a coherent system in which units that are correspondingly larger are multiples of smaller ones, building up the basic unit of linear time – the year. The cyclical view of time, on the other hand, refers to multi-year, recurring periods that arrange the history over the long term. The Bishop of Mende divides issues of time reckoning into 13 substantive chapters (caput) and a summary, characterizing the metric units of time, chronometric components and the order of the liturgical year. He also focuses the narrative of the treatise on the dual organization of the church calendar around the course of the sun and moon, the most important celestial bodies that form the basis of time reckoning.19 The chapters of the first part of Book VIII of Rationale address issues of definition, structure and functionality in the calendar of the solar year, month, week and day, centred around the movement of the Sun. The second part deals with the lunar year, lunar regulars, epacts, embolism, the golden number and determining the date of Easter. In the book’s conclusion, the author emphasizes the cyclical dimension of time and points to methods that allow a periodic approach to chronology, including ecclesiastical chronology.

Within the framework of the annual reckoning, the computist distinguishes 10 basic units of time measurement, each of which is a component of the hierarchi-

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17 ‘Compotus est computatio temporum secundum cursum solis et lune.’
19 Durand, Guillelmi Duranti Rationale I, 5: ‘Ceterum tria sunt tempora annorum sive triplex est annus videlicet Solaris, lunaris et Magnus. De qui bus previdimus in hac octavo parte tractare que duas principaliter habet partes in quorum prima agitur de anno solari et de magno, in secunda vero de lunari.’
cal pyramid of time. They are: year, month, week, day, quarter, hour, punctum, momentum, ounce and atom. He considers the year, month, week and day as components of the massa compoti. The remaining volumes, which are considered non-calendar, are not used in time reckoning, but only complement the whole in this categorization. Describing the temporal quantities used in chronology, the author begins by pointing out phenomena that are known, possible and easy to observe. The solar year, on the basis of which the liturgist forms the definition and rules of the year in the calendar, is the primary astronomical constant that regulates the succession of time. The author points out the differences in naming and distinguishing the different types of the year. However, the context of the narrative allows us to conclude that by referring to the term year or its different types (tempora anni),20 the scholar means the types of cycles that regulate the reckoning of time: the solar cycle, the lunar cycle and the grand cycle.21 Following the example of his computist predecessors, the Bishop of Mende establishes the cyclical nature of time using the canvass of his explanations of time reckoning. An allegorical depiction of this perspective is provided by the Egyptian snake that devours its own tail, Ouroboros, cited by the author.22

The periodic passage of time and the same nature of the year are emphasized, even more emphatically, by the characterization of the Zodiac belt as a kind of circular path of the Sun in its spectral journey across the sky.23 Each of the zodiacal signs is presented in the context of the distinctive characteristics of the creature whose name it bears, and these characteristics not only define a particular sign, but are translated into the behaviour and activity of the Sun entering the range of that sign. The broad context of each sign’s characteristics embeds it in a universalist vision of the universe, according to which earthly phenomena are reflections of heavenly realities, while the extraterrestrial world exhibits some characteristics of the material world. Here he juxtaposes astronomical and biological reality, making additional references to mythology. Take as an example the description of the sign

20 The term used here to mean different types of year can be confusing. This is because Durandus uses the term tempora anni to designate the different types of the year, customarily indicating seasons in chronology, rather than types of the year. It would be less ambiguous here to use the term tempora annorum, which would convey a number of diverse year types, rather than separate structures within a single year.

21 By grand cycle, the author of Rationale means the number of years resulting from multiplying the number of years of the solar cycle by the years of the lunar cycle: $28 \times 19 = 532$ years.

22 Durand, Guillelmi Duranti Rationale III, 1: ‘Sic apud Egyptians annus indicabatur: pingebant enim draconem caudam suam mordentem quia in Se recurrit et sic adhuc a gibusdam figuratur.’ The motif of a snake devouring its own tail as a representation of the cyclical nature of time appears in many medieval computi, including those of Bede the Venerable and Johannes de Sacrobosco.

23 Durand, Guillelmi Duranti Rationale III, 5: ‘Hec sunt signa poli que semper sunt via Solis: / est Aries, Taurus, Gemini, Cancer, Leo, Virgo, / Libraque, Scorpio, Chiron, Capricornus, Urmula, / Piscis.’ Durandus uses less common names for the two characters: Chiron – to denote Sagittarius (more commonly Sagittarius or Architenens) and Urmula, which is the lesser-known name for Aquarius (Aquarius, Amphora, Urna).
of Aries, whose influence on the activity of the Sun faithfully reflects the peculiar behaviour of the representative of this species in the animal world.\textsuperscript{24}

Analogies between the different planes of the universe provide a link between \textit{Rationale divinorum officiorum} and earlier cosmological treatises. The treatise clearly draws on the vision of the world and attempts to explain it given by Isidore of Seville and his monumental compendium \textit{Etymologiarum libri XX}. Like the Bishop of Seville, the Bishop of Mende characterizes the unknown elements of the cosmic world by comparing them to the closer and better known earthly reality. In this way, the liturgical treatise also acquires the characteristics of a universal work that conveys knowledge about the world. Referring to the naming and functioning of the seasons, the scholar proves that astronomical phenomena and the knowledge pertaining to them are within the reach of any keen observer of the surrounding world. In a similar context, he explains the names and roles of each month, with explanations linked to the framework liturgical calendar. Apparently, the distribution of the astronomical year is regulated in human consciousness with the help of the fixed dates of the liturgical calendar, whose various feasts open different periods of the astronomical year with their associated fasting periods.\textsuperscript{25}

\begin{quote}
Festum Clementis\textsuperscript{26} yemis caput est orientis,
Cedit yemps retro cathedrate Simone Petro,\textsuperscript{27}
Ver fugat Urbanus,\textsuperscript{28} estatem Symphorianus,\textsuperscript{29}
Id tibi quo restat autumpni tempora prestat.\textsuperscript{30}
\end{quote}

The observation and use of the periodicity of astronomical phenomena and the functioning of nature constitutes, as already mentioned, the basis of the system of time reckoning. The phenomenon of the cycle and its interpretation unifies the vastness of chronological knowledge and provides a common denominator to the basic and more advanced components of such a reckoning. The cycle has been integrated into the world and, without any deviation, regulates its history in

\begin{footnotesize}
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\item \textsuperscript{24} Durand, \textit{Guillelmi Duranti Rationale} III, 6: ‘Primum igitur signum est Aries, in quo sol dicitur esse factus, in quo signo est sol circa principia veris, sic dictum quia Sikut aries est Animals retro debile habend ante aliquid roboris, sic postrema solis illo tempore debilia sunt.’
\item \textsuperscript{25} Durand, \textit{Guillelmi Duranti Rationale} III, 21: ‘Vult Crux, Lucia, Cinis et Charismata dia / Ut det vota pia, quarta sequens feria.’
\item \textsuperscript{26} 23 November.
\item \textsuperscript{27} 22 February.
\item \textsuperscript{28} 19 May.
\item \textsuperscript{29} 22 August.
\item \textsuperscript{30} Durand, \textit{Guillelmi Duranti Rationale} III, 21. The verses indicate that the day of St Clement, or the eighth calends of December (23 November), marks the beginning of winter, which lasts until the Feast of the Chair of Saint Peter, or the eighth calends of March (22 February). Then spring begins and lasts until St Urban’s Day, or the eighth calends of June (22 May). That day marks the beginning of summer, which lasts until the memory of St Synphorian, the eighth calends of September (22 August). Autumn then begins and lasts until St Clement’s Day, which, again, marks the beginning of winter.
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an astoundingly repetitive manner. The author emphasizes six types of cycles: solar (28 years), 19-year (19 years), lunar (19 years), epactal (19 years), grand (532 years) and indictional (15 years). Like the ancient computists, he treats the lunisolar cycle as the basis of his chronology, which was used to construct the Paschal cycle and date the church festivals.

However, the scholar presents the definition and typology of the cycle at the very end of the description of the calculations, even after the instruction to determine the date of Easter in a given year. Such a procedure binds, in a peculiar way, and crowns the elements of the chronological account and also shows them in a unified context. This impression is reinforced by the author’s assignment of the closest related component to each cycle. By doing so, he further organizes the content of the treatise, pointing out their common basis in the reckoning of time. According to this grouping, the solar cycle is 28 years long and forms the basis of the concurrents and solar regulars, the letters of Sunday and leap and ordinary years. The 19-year cycle includes a golden number and a key number of chronological components (claves terminorum). The epactal cycle, which begins on the fifth calends of September, also lasts 19 years and constitutes the basis for calculating lunar regulars, epacts and ordinary and intercalary years. The grand cycle (532 years) illustrates the creation and operation of Dionysiac Paschal tables. Thus, the scholar emphasizes the fact that despite the complexity and richness of the elements and contents of the computational calculus under discussion, they all have their source, as well as their complement, in one and the same phenomenon – the phenomenon of periodicity in the world.

The system of computistic components

In Book VIII of the treatise, as mentioned above, two parts can naturally be separated. Issues in computing focus on the most relevant computistic components, their calculability and the use of computational results. Durand draws attention to the system of ferial letter, solar regulars, concurrents, lunar regulars, epacts,

31 Durand, Guillelmi Duranti Rationale XIII, 1: ‘Est enim ciclus vocatus eo quod in orbem dige-stus sit et quasi in circulum dispositum ordinem complectatur annorum sine varietate et sine arte.’

32 Durand, Guillelmi Duranti Rationale XIII, 4. The third type of cycle discussed, the lunar cycle, which, like the 19-year cycle, begins in January, also lasts 19 years, but its beginning is set in the fourth year of the 19-year cycle. Durandus claims it was invented by the Romans when they discovered the time at which the moon brightens from the sunlight (‘qua hora luna a sole accende-retur’). However, since this invention has become erroneous and difficult to interpret over time, as the scholar concludes, it does not form the basis for any calculations.

33 The 19-year cycle, the lunar cycle and the epatical cycle are each 19 years long, but differ in the date of their beginning. This is because the lunar cycle begins in the fourth year of the 19-year cycle, while the epatical cycle begins on the fifth calends of September (August 25).

34 With the help of concurrents, it was very easy to determine the day of the week for the first day of the year. This was done in such a way that the first seven digits were assigned the names of consecutive days of the week, starting with Sunday. Thus, 1 = Sunday, 2 = Monday, 3 = Tuesday, 4 Wednesday, 5 = Thursday, 6 = Friday, 7 = Saturday. For technical reasons, the computists, designating the days of the week by ecclesiastical names, called all these days feria, i.e. dies dominica = feria prima, Monday = feria secunda..., sabbatum – feria septima. See Opera, cap. 4, p. 95 et seq.
embolism, the golden number, the date of Easter and the calendar of movable feasts regulated by it. The scholar also found it necessary to explain the meaning and functioning of the cycle as one of the basic constants of orientation in time. The ecclesiastical computus (computus ecclesiasticus), also known as the ordinary or common computus (c. vulgaris), is based on a collection of these components. When using the lunisolar computus, there was a need to perform calculations and computations to determine the place of a given year in the next year of the cycle, usually lunar or solar. The scholar points out in the context of each calculation that the diversity of operations is due to differences in the reckoning of solar and lunar years, which in turn translated into the need for two separate systems of regulars.

The second part of Book VIII of the work is an abridged textbook of astronomical and chronological mathematics. It presents formulas and algorithms to determine the individual components of the lunisolar calendar, albeit these are not only calculations for selected constants, but a comprehensive calculation of the components according to the variables. To simplify the reckoning, he also uses examples of calculations employing a chirometric computus to make calculations easier, as in the case of the claves terminorum.35

The basis for the calculation of regulars36 in the lunar cycle is to outline the definition of the lunar year, which, like the solar year, is defined in three ways. A lunar year can be defined both as the time when, analogous to the Sun, the Moon travels through the entire area of the Zodiac belt; by another definition, it is the time the Moon circles the Sun between two full Moon phases, whereby it would be more appropriate to speak of a lunation or lunar month. The scholar also points out that the lunar year can include 12 or 13 lunations.37

Based on the data of the lunar year, the computist comes up with a series of calculations of the regulars, epacts, concurrents and date of Easter. In addition to sample numerical calculations, he also uses mnemonic aids – short, one-verse or several-verse rhymes that make it easier to learn the principles of mathematical calculus. When defining lunar regulars, the author uses verse:

Dic e, g bis, i k bis, et post manet ordo38

35 Durand, Guillelmi Duranti Rationale XII, 2: ‘Invento quotus est annus clavium, possumus scire que sit clavis illius anni per hanc aliam rationem; numerus enim ille qui remanet de divisione annorum Domini numerari debet per summitates quinque digitorum, inchoando a pollice, et si numerus terminatur in pollice, debemus addere viginti quinque, et summa inde proveniens erit clavis illius anni (…).’

36 Regulars were fixed numbers to be memorized. Combined with the variable elements of lunisolar reckoning, they allowed the ratio of days of the week to days of the civil and lunar months to be determined for a given year.

37 Depending on whether it is a normal year or an embolismic year.

38 According to W.E. van Wijk, the same sequence of syllables referring to the lunar regulars is found in Dulciatus, while Alexander of Villedieu in Massa compoti 290–295 gives a sequence of lunar month regulars beginning with September: (5, 5, 7, 7, 9, 10, 9, 10, 11, 12, 13, 14). Variants of these calculations can also be found in John Sacrobosco: ‘Quinque Sep. Oc. Datur, No .De . septem, ter, tria la . Mar / Feb . A . decem sumant, post unum cuilibet addas.’ Wijk attributes the expanded version to Byrthferth:
Each letter was assigned a numerical value, corresponding to the order of the letter in the alphabet. Calculating the meaning of this verse should be done in two ways, because on the one hand, the order of the lunar months must be kept in mind, and on the other, the order of the numbers corresponding to specific letters of the verse must be maintained. The calculation results in a fixed number, assigned to each month to determine the age of the moon on the first day of the month.

The epactal system can also be used to determine the age of the moon on the first day of the month, which is included as supplementary material to the knowledge of lunar calculus. In its definition, epacts are variable numbers assigned to particular years, which are also used to determine the age of the moon on the first day of a given month. This variable is more complex than the lunar regular, because it takes into account the difference in the length of the solar and lunar years, and the number of epacts is due precisely to the difference of 11 days in the duration of these types of years. For proper calculation of epacts, a triple modus operandi can be used, consisting of: numerical calculus, chirometric calculus and mathematical verse, which includes an abbreviated paradigm for epact calculus. Only when the number of lunar regulars and epacts are added together do we obtain the actual age of the moon on the first day of the month.

The epactal system is based on the difference of 11 days between the duration of the lunar and solar years. Thus, when calculating the number of epacts, one must remember to add the number 11 in successive years, making sure that the sum does not exceed 30. The epact calculus for the 19-year cycle thus closes in a regular sequence: 1 – nulla, 2 – 11, 3 – 22, 4 – 3, 5 – 14, 6 – 25, 7 – 6, 8 – 17, 9 – 28, 10 – 9, 11 – 20, 12 – 1, 13 – 12, 14 – 23, 15 – 4, 16 – 15, 17 – 26, 18 – 7, 19 – 18. It should also be noted that the use of the golden number in lunar cycle calculations has supplanted the epactal system; however, Rationale mentions both those systems.

The cycle of epacts and lunar regulars is not strictly repetitive due to the phenomenon of embolism, which the author, following the example of his predecessors and contemporaries, calls the lunar leap (saltus lunae). Embolism involves the occurrence of 13 instead of the usual 12 lunations in a year, which also translates

September semper quinis October habenis.
Sicque November ouat septenis atque December.
Janus rite nouem sic Mars retenat eosdem,
Inde decim Februus, regulator Aprilis et isdem
Hinc Maius comites undenos ordine querit.
Iunius exulantque retetat iure duodenos.
Iulius At Cesar tredenis testibus instant.
Quartenis regularibus at letatur Augustus.
Hos igitur regulares si coniungas epactis
Cognosces mensium qua luna etate kalendas,
Quolibet he numero ultra sint citrave trigena.


into a difference in the number of days – an ordinary year of 12 lunations has 354 days, while an embolismic year is 30 days longer (with a total of 384 days). Such a leap is attributed to the 7 years of the lunar cycle and regulates the differences from the solar cycle in such a way that in the 19-year lunar cycle there are 12 ordinary years and 7 embolismic (extended) years. The scholar concludes the complicated sequence of necessary calculus by presenting the possibility of verifying the obtained results using other calculus methods and adopting other variables.

Following the example of the ancient computists and his later predecessors, and presenting the necessary methods of calculation and ways to verify them, Durand draws attention to the imperfection of reckoning and the inaccuracies that result from it – such as the omission of one day of the year despite the ongoing lunation on that day. As the most reliable and accurate corrective tool, he presents one of the most basic and widespread computistic components – the golden number (aureus numerus), which indicates the first day of the lunation for each month. The chronological invention of Julius Caesar (according to the tradition also followed by Durand) surpasses and combines all the other components of chronometric reckoning, since it makes it possible to determine the first day of the lunation for any year of the lunar cycle. The third day of the lunar cycle falling on the first day of January is taken as the starting point of the calculation, and the cycle of

40 Durandus also provides an interesting rationale for the naming of the ordinary and embolismic year. An ordinary year (communis) owes its name to its frequency of occurrence (it is possible for two ordinary years to occur in a row), which cannot be the case with an extended year, as such a year always occurs as singular. Durand, Guillelmi Duranti Rationale XII, 3: ‘Et dicitur annus communis quia sepe duo ita coniunctim incedunt ut invicem se in paschali solemnitate sequantur, embolismalis vero semper solus est.’

41 The 19-year cycle is often referred to as the Metonic cycle, since its creator was Meton of Athens (d. 460 BC). He calculated that after 19 solar years of 365 days and 6 hours each, the moon returned on the same days it had been 19 years before, and that 235 lunar months had passed in that time period. These 235 lunations resulted in a period of 19 years, of which 12 years involved 12 lunations each and seven years – 13. The latter were called leap, intercalary or embolismic years.

42 According to Wijk, this entire cycle contains the Ogdoad and the Endecad. Citing Dionysius: ‘noverimus itaque quia idem decemnovennalis cyclus per ogdoadem et endecadem semper in se revolvitur’ (PL 67, 513). The Ogdoad, which was too short by one day, begins the day before the 1st of Toth, and ends on the 1st of Toth; the Endecad that, on the contrary, is one day too long, begins the day after the 1st of Toth and ends the day before the 1st of Toth. Consequently, the Ogdoad has, except for leaps, $8 \times 365 + 2$ days, and the Endecad has $11 \times 365 - 2$ days, for a total of 6935 days. When one inserts lunations alternatively for 30 and 29 days, and when one lunation is assigned to each solar month, one needs $19 \times 6 = 114$ rows of 2 lunations of 59 days each, a total of 6726 days, and from the remaining 209 days 7 additional months, called embolismic months, are created, which are included in the cycle at the appropriate places. Embolismi (embolismic months) are all 30 days long and one day is removed, reducing the last cycle to 58 instead of 59 days. This missing part of one day is called the lunar leap saltus lunae, which intrigued medieval computists. See Wijk, Le nombre d’or; idem, Onze kalender, Amsterdam 1955, pp. 44–45.

43 Durandus reiterates this view after Alexander of Villedieu; the latter is known as the computist who first used the term aureus numerus. However, modern researchers do not agree on this. It is certain, however, that he lived and worked at a time when computational studies were being diligently practised, and the term golden number appears precisely in Alexander of Villedieu’s treatise
The golden number is therefore formed on the basis of this third day of the cycle falling on the first day of January.

The author emphasizes the importance of the golden number, granting it primacy over all other lunar components and comparing it to gold, which is superior to all other bullion in nobility, just as aureus numerus towers over the other lunar numbers. It should be noted that he is one of the first western computists to proclaim the perfection of this number, for he is almost a direct continuation of the chronological thought of Alexander of Villedieu (d. 1240), who grafted the concept of the golden number onto the ground of European computist studies.

When calculating the age of the Moon on the first day of the month, it is also worth pointing out the relationship of the days of the week to the days of the month. The first day of the month falls on a day of the week according to the rules of reckoning of the 28-year solar cycle, whose successive years are counted using the first seven letters of the alphabet A-G. To demonstrate the truth of this thesis, the scientist uses two lines. The first one:

Alto domat Dominus, gratis beat, equa gerentes,
Contempnit fictos, augebit Dona fideli.

consists of 12 words, corresponding to 12 months: the first one to January, the second one to February and so on, according to the succession of months. Furthermore, the author assumes that the initial letter of each word determines the letter of the calends (first day) of the month to which the word is assigned. For example, the first word is alta and it refers to the first month, January, and its first letter is a. A is therefore the first ferial letter for January. The second word – domat – refers to the second month, February, and its first letter is d. It is therefore the first ferial letter of February. Subsequent analogous procedures will demonstrate the correctness of this assumption. They are also illustrated in a verse composed of dictiones, or syllables:

Adam degebat, ergo cifos ad rifex

Massa compoti. Attributing the invention of this component to Julius Caesar may have been intended to make its reception easier and faster. See Wijk, Onze kalender, p. 68 et seq.

44 Based on the calculations of the Athenian Meton, medieval computists created an algorithm for calculating the lunar months and years of the 19-year cycle in conjunction with the components of the solar calendar, which was called the golden number (aureus numerus). According to many researchers, it is the most important invention of the chronological culture of the Middle Ages, while the adjective aureus emphasizes the special importance of this number in computistics.


46 Ferial letters.

47 This line is one of the more frequently cited by computists, probably intended for beginners in computational calculus, sometimes additionally written with majuscule highlighting significant letters in syllables: ADam DeGeBat ErGo CiFos AdriFex. The same content is given in the computistics in a form that is even more accessible to the reader, i.e. in the breakdown of the entire composition into single significant syllables: A . dam . de . / ge . bat . / er . go / ci . fons / a . de / fe . lix. Lix is the 13th syllable in this chronogram; as such, it cannot exist on its own, which is why it was placed next to fe. The formal
As far as the algorithm is concerned, knowing the use of the above verse and the letter of calends, which is the first ferial letter of the month, as well as the calculated Dominical letter for the year, one can easily determine the day of the week on which the month begins. The order by Sunday (Dominical) letter of the year is then adopted, counting the days by successive letters up to the letter that falls on the first day of the month whose first day of the week we designate. The day it occurs on is the first day the month.

Thus, it presents an abbreviated yet comprehensive manual of the reckoning of the days of the week and the month, placing them in a mutual relationship of dependence and illustrating their functioning in dependence on the age of the moon. The computist avoids long, syllogistic calculations, focusing instead on discussing specific examples for each type of calculation.

Calculating the date of Easter

The system of the components of time reckoning in the work of the Bishop of Mende is crowned by a concise guide to calculating the date of Easter using the auxiliary numbers discussed earlier. The matter, which was the subject of deliberations and treatises of his predecessors (Dionysius or Alexander of Villedieu), forms part of a broad liturgical reality in Rationale. In this way, the principles of computational reckoning and Paschal calculations become part of the liturgical regulations and begin to function in legal terms.

Collecting the findings of his predecessors, the author points out that calculating the date of Easter can be done in three ways: using the system of claves terminorum, using the age of the moon, and using the Paschal tables. Clavis terminorum is a variable number that is used to determine the first of the five dates of the movable holidays, which are closely related to the date of Easter: Shrovetide, Lent, Easter, the Days of the Cross and Pentecost. In other words, following the key-number, the first date opens the sequence of holidays that depend on the date of Easter.

The mathematical structure of claves terminorum is based on the 19 years of the lunar cycle and the 7 days of the week, whose interdependence makes it possible to calculate the dates of the landmark points of the liturgical year. The association of the lunar cycle with claves has been known since the early 13th century. In order to make this chronological category, which is new to the tre-
atise, easier to assimilate, the computist recalls the concepts of epacts and lunar cycle, which are already familiar to the reader from the earlier arguments – and not only recalls them, but points out the analogies and differences between them. The building of awareness by the scholar about the fact that different components can be calculated using the same formulas consolidates and unifies the arithmetic system he constructs. In other words, the technique of reckoning based on claves terminorum combines two elements of the lunisolar calendar: consecutive years of the 19-year cycle with a specific number of days assigned to them. Claves terminorum in the 19-year cycle can be presented in the following order: 1 – 26, 2 – 15, 3 – 34, 4 – 23, 5 – 12, 6 – 31, 7 – 20, 8 – 39, 9 – 28, 10 – 17, 11 – 36, 12 – 25, 13 – 14, 14 – 33, 15 – 22, 16 – 11, 17 – 30, 18 – 19, 19 – 38.50

By employing claves terminorum, one can enumerate all the dates of movable feasts associated with Easter, and each separate key-number will indicate the date of a separate feast. A simpler method, according to the scholar, seems to be to use the age of the moon as a starting point for calculating the first key date – the Shrovetide (Septuagesima). This is because knowing the daily date of this feast and knowing the time distances between successive festa, one can easily arrange their chronological sequence. The basis for further calculations in this case is the age of the moon on the day of Epiphany (Circumcisio Domini). After calculating, according to Jewish tradition, the age of the moon by successive calendar days by 40 or 41, the following verses quoted by the scholar become a further manual of calculation:

A festo stelle51 numerando perfice lunam
Quadraginta dies post Septuagesima fiet.
Bissextus quando fuerit, superadditur unus.
Si cadit in lucem Domini, tunc sume sequentem,
Si cadit in feriam septenam, sitque bissextus,
Linque diem Domini primum, sumasque secundum.

In other words, claves terminorum – the key-numbers of the dates (Septuagesime, Quadragesime, Pasche, Rogationum, Pentecostes) are the numbers that make it possible to determine the Easter boundary without a perpetual lunar calendar. Claves set (in days) the distance of an arbitrary day (March 11) from the corresponding Easter boundary (terminus paschalis). Thus, certain dates were also set for the Sundays from which, counting forward with the number of claves, one arrived at the paschal date terminus paschalis. The following Sunday was the sought-after feast. The Sunday after the Easter border was Easter Sunday.

From this reckoning, it was easy to extend the claves to other movable feasts of the Easter cycle. There were five such claves, also known as locus clavium or sedes clavium: 7 January – clavis septuagesime, 28 January – clavis quadragesi-
me, 11 March – clavis pasche, 15 April – clavis rogationum and 29 April – clavis pentecostes.

From these fixed key dates counted forward the next feast dates were derived. The following Sunday was the sought-after feast. The number of claves terminorum expressing the distance between ‘seats’ of the key-numbers with the sought-after feast was valid only for a specific year as shown in the following list, which gives the numbers of regulares clavium.

<table>
<thead>
<tr>
<th>Claves</th>
<th>26</th>
<th>15</th>
<th>34</th>
<th>23</th>
<th>12</th>
<th>31</th>
<th>20</th>
<th>39</th>
<th>28</th>
<th>17</th>
<th>36</th>
<th>25</th>
<th>14</th>
<th>33</th>
<th>22</th>
<th>11</th>
<th>30</th>
<th>19</th>
<th>38</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. golden</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td>8</td>
<td>9</td>
<td>10</td>
<td>11</td>
<td>12</td>
<td>13</td>
<td>14</td>
<td>15</td>
<td>16</td>
<td>17</td>
<td>18</td>
<td>19</td>
</tr>
</tbody>
</table>

The key-numbers of the year were closed in a 19-year cycle, as the Passover dates are calculated according to the lunar calendar.\(^{52}\)

The determination of the date of Easter is also explained by the author using the example of a (chirometric) manual computus, the usefulness of which he mentions, but omits a detailed analysis for the sake of consistency and clarity of the argument. He cites the Paschal reckoning of Dionysius Exiguus and Isidore of Seville as the most famous creators of Easter tablets. The declaration of preserving the heritage of the fathers of European computistics legitimises the vision of the Rationale author’s own computus and emphasizes the existence of a long tradition of time reckoning, especially the Paschal reckoning.

**Conclusion**

The observation of the relationship between the periodicity of astronomical and terrestrial phenomena, as well as the related and consequent marking and reckoning of time, has become a field of knowledge practised by a very small group of specialists called computists, drawn from ecclesiastical circles since ancient Christianity. The treatise of the Bishop of Mende also has a pastoral dimension.\(^{53}\) By presenting astronomical and chronological phenomena, Durandus attempted to bring the algorithm of the ecclesiastical computus and time reckoning to a wider audience. Among other things, its creation was the result of concern for pastoral care in the diocese. He was the first to recognize computational chronology as an integral part of legal doctrine. The creation of the textbook, which addressed the widely discussed issues related to the liturgy, was aimed at explaining the importance of the various places, rites, and objects used in the liturgy and setting all these elements in the rhythm of the church calendar. His treatise is not just

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\(^{52}\) For example, for Easter of the year 1277, the golden number is 5 and the key-number is 12. Counting 12 days from March 11 (claves pasche), Easter falls on 22 March that year. To calculate the days of the week in relation to the days of the month for each year, a concurrent was used, and after the invention of ferial letters, it was counted according to the Dominical letters.

a codification of pastoral liturgy, but a kind of manual of ecclesiastical *computus ecclesiasticus*.\(^{54}\)

The presentation of content on technical chronology, as the author himself states, is neither exhaustive nor innovative. Durandus admits that his treatise is a compilation of the thoughts of other commentary authors and works on liturgical, legal and computational topics. He elaborates on the difficult parts of the computus, citing the findings of these predecessors. The author attributes the non-exhaustive dimension of the work to human frailty and the impossibility of capturing the enormity of creation in such a content-diverse treatise. The creative process, based on the thought of other scholars, is shown by the Bishop of Mende on the plane of allegory, comparing the work of the liturgist and the computist to the toil of the bee, gathering nectar from many flowers.\(^{55}\) Durandus’ work follows the approaches of earlier computists, whilst its originality lies in the choice of content. He presents some chronological issues at length (like the 19-year cycle or months). Others, however, are only signalled (such as the issue of concurrents or *claves terminorum*, which needed to be developed in the body of the article).

The concise concept of the ecclesiastical computus contained in *Rationale divinorum officiorum* refers in content to the findings of the computists of the Roman reckoning of Easter. The author mentions Isidore and Dionysius Exiguus; there is also apparent inspiration from the work of Alexander of Villedieu, whose author of *Rationale* is almost a direct successor in chronological matter. His treatise, while setting forth rules for dealing with liturgical matters, also consolidates and popularizes the Roman system of Paschal reckoning following his predecessors, and defines the ecclesiastical computus as an essential element in the operation of canon law.

*Translated by Marek Robak-Sobolewski*

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\(^{54}\) Ibidem.

\(^{55}\) Durand, *Guillelmi Duranti Rationale* XIV: ‘Hoc enim tam ex diversis aliorum libellis et commentariis more mellificantis apis, quam ex hiis que mihi divina gratia propinavit fructuose collegi, et hanc doctrinam interno fluentem nectare, velut favum mellis in divinis officii speculare volentibus (…)’.
THE DURAND COMPUTUS IN BOOK VIII RATIONALE DIVINORUM OFFICIORUM


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KOMPUT DURANDUSA W VIII KSIĘDZE RATIONALE DIVINORUM OFFICIORUM

Streszczenie

Zasadniczym zadaniem opracowania jest przedstawienie narzędzi komputystycznych, którymi posługiwał się Durandus. Czas jego aktywności naukowej przypada na wczesny okres rozwoju komputystyki, kiedy poszczególne kom-

Słowa kluczowe: Durandus; chronologia średniowieczna; komput; rachuba paschalna; czas liturgiczny